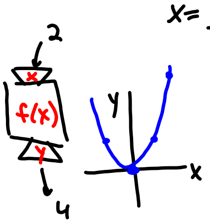
10.1 Parametric Curves

function: y=x^2

$y = f(x)$

indep dep

x	y
2	4
3	9
-2	4
0	0



parametric: x=t, y=t^2

$x = g(t) \quad y = h(t)$

t	x	y
2	2	4
3	3	9
0	0	0
-2	-2	4

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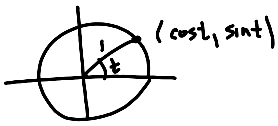
x=cos(t)
y=sin(t)



x=3cos(t)
y=sin(t)

$x = \sqrt{t}$
x=sqrt(t)
y=t-2

$t = x^2$

$y = x^2 - 2, \quad x \geq 0$

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slope of a parametric curve

$$\boxed{\text{slope} = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}}$$

$$\frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

second derivative

$$\boxed{\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}}$$

Feb 20-8:01 AM

Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

$x = \cos t$
$y = \sin t$

$$\frac{dy}{dx} = \frac{\cos t}{-\sin t}\Bigg|_{t=\frac{\pi}{4}} \rightarrow \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(-\sin t)(-\sin t) - \cos t(-\cos t)}{(-\sin t)^2}}{-\sin t}$$

$$\frac{(\sin^2 t + \cos^2 t)}{(-\sin t)^2} \cdot \frac{1}{-\sin t} = \frac{1}{(-\sin t)^3}\Bigg|_{t=\frac{\pi}{4}}$$

$$\frac{1}{\left(-\frac{\sqrt{2}}{2}\right)^3} = \frac{-2^3}{\sqrt{2}^3}$$

Feb 7-10:06 AM

$$\boxed{\text{arclength} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt}$$

Feb 7-10:18 AM

find the length of the curve x=cos^3(t) y=sin^3(t)

$x = \cos^3 t \quad y = \sin^3 t$

astroid

$$4 \int_0^{\pi/2} \sqrt{(-\sin t \cdot 3\cos^2 t)^2 + (\cos t \cdot 3\sin^2 t)^2}\, dt$$

$$4 \int_0^{\pi/2} \sqrt{9\sin^2 t \cos^4 t + 9\cos^2 t \sin^4 t}\, dt$$

$$4 \int_0^{\frac{\pi}{2}} \sqrt{9\sin^2 t \cos^2 t\,(\cos^2 t + \sin^2 t)}\, dt$$

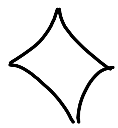
$$4 \int_0^{\pi/2} 3\sin t \cos t\, dt = 4\cdot 3\left[\frac{\sin^2 t}{2}\Big|_0^{\pi/2}\right.$$

$u = \sin t$

$du = \cos t\, dt$

$$= 12\left[\frac{1}{2} - 0\right]$$

$$= 6$$



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find the length of one arch of the cycloid x=a(t-sin t), y=a(1-cos(t))

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